REVIEW

Studies of mixed convection in vertical tubes

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The early study of convective heat transfer considered the branches of forced and free convection independently with only passing reference to their possible interaction. In fact the two are extreme cases of the general condition of "mixed" or "combined" forced and free convection where both mechanisms operate simultaneously. The present contribution aims to provide an up-to-date review of those works concerned with mixed convection heat transfer in vertical tubes. The review is divided into two sections, the first dealing with laminar flow, and the second with turbulent flow; further subdivisions are made according to whether the work is theoretical or experimental. Comparisons between theory and experiment are made where possible, expressions defining the conditions for onset of buoyancy effects are presented and equations for determining heat transfer are given. The paper ends with some general comments and recommendations. The survey is restricted to fluids of moderate Prandtl number; mixed convection in liquid metals can display very different characteristics which will be discussed in a future paper.

Keywords: mixed convection; combined convection; buoyancy-influenced flow; interaction between forced and free convection; laminarization; thermogravitation

Introduction

The term "mixed convection" is used to describe the process of heat transfer in fluids where, due to variations of gravitational body force associated with non-uniformity of density within the system, the flow field is significantly modified from that which would prevail under conditions of uniform density. The processes involved are usually thought of in terms of the concept of fluid buoyancy and the effects are frequently referred to as effects of buoyancy on heat transfer.

In the early development of the subject of convective heat transfer, free and forced convection were studied separately and any interaction between the two was ignored. When the possibility of such interactions began to be investigated, attention was at first restricted to laminar and transitional flow conditions. More recently it has become clear not only that measurable influences of buoyancy can exist in fully turbulent flow, but that under certain conditions buoyancy effects can in fact be the dominant factor in determining heat transfer.

In this paper attention is focussed on mixed convection in vertical tubes. Two thermal boundary conditions are of particular interest, namely uniform wall temperature and uniform wall heat flux. The early experimental work on mixed convection was carried out using test sections heated by means of saturated steam, resulting in approximately uniform wall temperature. Later workers have in the main utilized electrical heating leading to approximately uniform wall heat flux.

The effects of buoyancy on heat transfer rates can be either to enhance the process or to impair it depending on the flow orientation (ascending or descending), the flow conditions and on heated length. A sound understanding of the processes involved is needed in order to take proper account of the effects of buoyancy on heat transfer in the design of thermal systems.

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Laminar flow mixed convection

The effect of simultaneous buoyancy forces and externally-applied pressure forces on steady laminar flow in a vertical pipe is amenable to calculation. The general result is that when the flow is in the upward direction past a heated surface (or downwards past a cooled surface) heat transfer is enhanced, whereas in the opposite cases heat transfer is impaired. These influences are not the result of any change in thermal diffusivity but are instead a consequence of the distortion of the velocity field and pattern of convection in the fluid.

Theoretical studies of laminar mixed convection

Work on the subject dates from 1942, when Martinelli and Boelter[1] analysed a very simple fully developed flow model. Other fully developed solutions were subsequently reported by Ostroumov,[2] Hallman,[3] Hanratty *et al.*,[4] Brown,[5] and Morton.[6]

With time and the advent of digital computers the number of the simplifying assumptions decreased and, in particular, attempts were made to obtain developing flow solutions. Rosen and Hanratty,[7] following earlier work by Pigford,[8] used the boundary layer integral method with power series for the velocity and temperature profiles. Thus they reduced the problem to one of integrating a number of simultaneous non-linear differential equations. Numerical solutions, taking account of the variations of all the physical properties, were obtained for upward flow of air with uniform temperature by Bradley and Entwistle.[9] Marner and McMillan[10] also obtained numerical solutions for this boundary condition taking account of flow development. Their predictions of arithmetic mean Nusselt number are shown in Figure 1. Figure 2 shows an interesting calculation of local Nusselt number for which the influence of buoyancy increases following the thermal entry development and then reduces further downstream as the fluid temperature approaches the wall temperature.

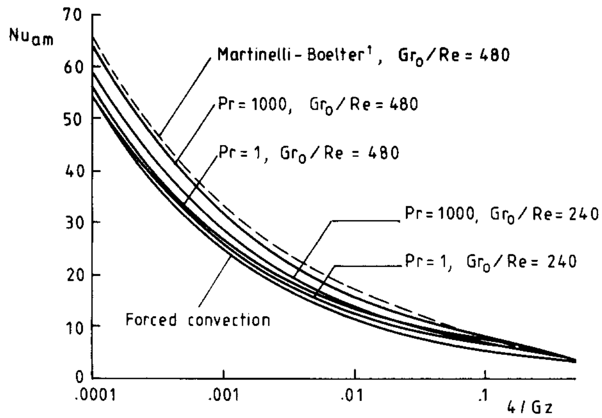


Figure 1 Comparison of arithmetic mean Nusselt numbers with the results of Martinelli and Boelter (adapted from Marner and McMillan[10]; with permission of the American Society of Mechanical Engineers)

Lawrence and Chato[11] and Collins[12,13] have developed numerical models for predicting developing mixed convection taking account of viscosity and density variations and using marching solution procedures. Zeldin and Schmidt[14] solved the full elliptic equations governing the problem using an iterative method in order to avoid the use of marching procedures, which had been questioned for conditions of strong

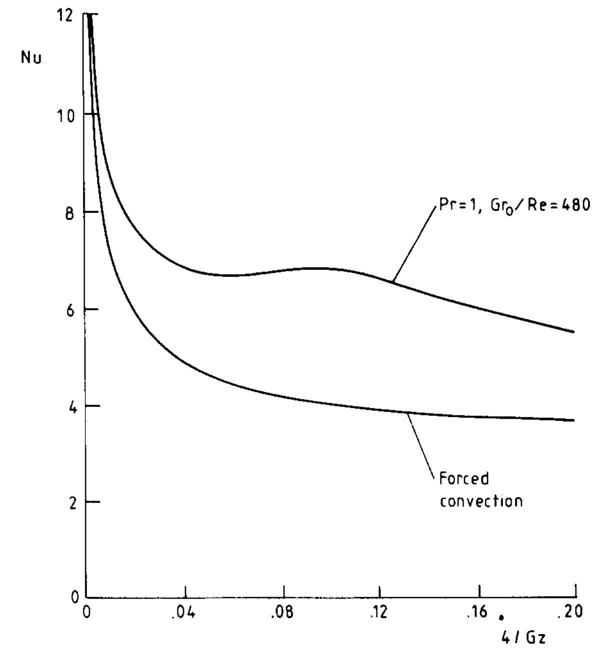


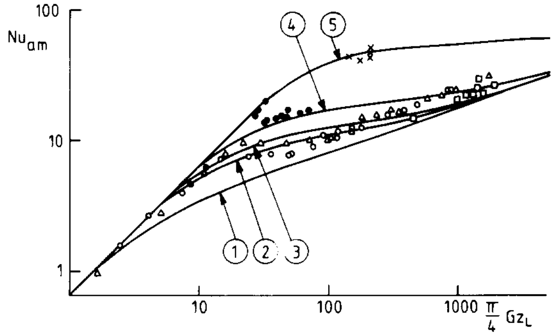
Figure 2 Typical behavior of local Nusselt number near the point of maximum velocity profile distortion (adapted from Marner and McMillan[10]; with permission of the American Society of Mechanical Engineers)

Notation

a	Radius of tube, $d/2$ (m)
C_p	Specific heat at constant pressure (J/kgK)
$\bar{C}_p$	Integrated specific heat, $\dfrac{1}{(T_w - T_b)}\displaystyle\int_{T_b}^{T_w} C_p\,dT$ (J/kgK)
C_μ	Coefficient in 2 equation turbulence model
d	Tube diameter (m)
g	Acceleration due to gravity (m/s^2)
G	Mass velocity, $4\,M/\pi d^2$ (kg/m^{2}s)
Gr	Grashof number, $(\rho_b - \rho_w)\,d^3\,g/\rho_b^2$
$\overline{\mathrm{Gr}}$	Grashof number based on $\bar{\rho}$, $(\rho_b - \bar{\rho})\,d^3\,g/\rho_b^2$
Gr*	Grashof number based on heat flux, $g\beta\,d^4 q/\lambda v^2$
Gz	Graetz number, Re Pr d/z $(=1/z^*)$
h	Heat transfer coefficient, $q_w/(T_w - T_b)$ (W/m^2 K)
i	Specific enthalpy (J/kg)
k	Turbulent kinetic energy (m^2/s^2)
l	Length scale (m)
L	Tube length (m)
M	Mass flow rate (kg/s)
Nu	Nusselt number, hd/λ
Nu_{am}	Arithmetic mean Nusselt number $\dfrac{\left\{\dfrac{1}{z}\displaystyle\int_0^z q_w\,dz\right\}D}{\lambda(T_w - \frac{1}{2}(T_o + T_b))}$
p	Pressure (N/m^2)
Pe	Peclet number, Re Pr
Pr	Prandtl number, $\mu C_p/\lambda$
$\overline{\mathrm{Pr}}$	Prandtl number based on $\bar{C}_p$, $\mu\bar{C}_p/\lambda$
q_w	Heat flux at the wall (W/m^2)
r	Radial coordinate (m)
Ra	Rayleigh number, Gr Pr
Ra*	Rayleigh number based on heat flux, Gr* Pr
Re	Reynolds number, Gd/μ
T	Temperature (°C or K)
T'	Temperature fluctuation (K)
v	Radial velocity fluctuation (m/s)
V	Radial velocity (m/s)
w	Axial velocity fluctuation (m/s)
W	Axial velocity (m/s)
y	Transverse coordinate measured from wall, $a - r$ (m)
y^+	$y\sqrt{\tau_w/\rho}/v$
z	Axial coordinate (m)
z^*	Dimensionless axial coordinate, $\dfrac{z}{d\,\mathrm{Re\,Pr}}$
β	Coefficient of volume expansion (K^{-1})
ΔT	Wall to bulk temperature difference, $T_w - T_b$ (K)
ε	Rate of dissipation of k (m^2/s^3)
κ	Von Karman constant
λ	Thermal conductivity (W/mK)
μ	Dynamic viscosity (kg/ms)
v	Kinematic viscosity, μ/ρ (m^2/s)
ρ	Density (kg/m^3)
$\bar{\rho}$	Integrated density, $\dfrac{1}{(T_w - T_b)}\displaystyle\int_{T_b}^{T_w} \rho\,dT$ (kg/m^3)
σ_t	Turbulent Prandtl number
τ	Shear stress, N/m^2

Subscripts

b	Refers to fluid bulk conditions
c	Refers to thermodynamic critical pressure
F	Forced convection
m	Mean or mixing length
min	Minimum
pc	Refers to pseudocritical value
ref	Reference value
t	Turbulent
w	Refers to conditions at the tube wall
z	Based on axial length
0	Refers to inlet conditions
∞	Refers to the fully developed condition



Equation (1)	$Gr_0 Pr\ d/L$
①	0
②	10^4
③	2×10^4
④	10^5
⑤	4.5×10^6

Experiment	$Gr_0 Pr\ d/L$
○	1.05×10^4
△	2.13×10^4
□	3.1×10^4
●	1.05×10^5
×	4.5×10^6

Figure 3 Experimental and predicted magnitudes of Nu_{am} as a function of Gz_L (adapted from Martinelli *et al.*[19]; with permission of the American Institute of Chemical Engineers)

buoyant influences. Nevertheless, difficulties in obtaining a solution were still encountered beyond a certain value of Gr/Re corresponding to a reversed flow in the central region of the pipe.

In the case of buoyancy-aided convection, i.e. upward flow in a heated pipe or downward flow in a cooled one, the velocity in the vicinity of the wall increases with wall-to-fluid temperature difference (at constant flow rate) and decreases in the core. Eventually a concavity develops in the velocity profile in the core flow. Associated with these changes there is an enhancement of heat transfer coefficient. In practice, with sufficiently strong heating or cooling, the velocity profile gradually becomes unstable at the point of inflexion and flow unsteadiness results (see below).

In the opposite case, i.e. downward heated flow or upward cooled flow, the velocity near the wall is reduced and there is an impairment of heat transfer. With strong enough heating (or cooling), the velocity gradient at the wall approaches zero and an instability develops there suddenly.

The numerical solutions of Collins[13] illustrate the above behavior. These calculations were carried out for conditions covered in the experimental work of Scheele and other workers on the influence of heat transfer on stability of laminar flow in vertical tubes.[11,15–17] It was observed that as Reynolds number decreased, the critical value of Gr*/Re for onset of gradual instability decreased asymptotically to approximately 42.5 for the case of upward flow with uniform heat flux. An increase in the critical value of Gr*/Re with decrease in Reynolds number was observed for the case of downward flow with uniform heat flux. The highest critical value of Gr*/Re encountered was approximately 75 and this was for Reynolds numbers approaching zero.

Experimental studies of laminar mixed convection in vertical tubes

The early experimental work involving mixed convection in vertical tubes was carried out using test sections in the form of double tube heat exchangers. Watzinger and Johnson[18] in 1939 reported experiments in which water flowing downward in a tube was cooled by an external flow. For that arrangement free convection aided forced convection. Martinelli *et al.*[19] shortly afterwards reported experiments with upward flow of water and oil in tubes having uniform wall temperatures, heated by means of steam. Overall heat transfer rates considerably in excess of those predicted for forced convection were observed (Figure 3). The approximate theoretical model of Martinelli and Boelter[1] was used with remarkable success as a basis for correlating the data. The following equation, which is identical with the theoretical result, apart from the slight adjustment of an index from 0.75 to 0.84, correlates all their data and that of Watzinger and Johnson to within $\pm20\%$.

$$Nu_{am}=1.75F_1\left\{Gz_m+0.0722F_2\left(\frac{Gr\,Pr\,d}{L}\right)_w^{0.84}\right\}^{1/3} \qquad (1)$$

F_1 and F_2 are functions which are derived from theory and tabulated in Ref. 1. No data were produced by Martinelli *et al.* for the case where free convection opposed forced convection (heated downward flow).

Subsequent work on laminar vertical mixed convection has been carried out using uniformly heated tubes. Clark and Rosenhow,[20] in experiments on water at high subcritical pressure flowing upwards in an electrically heated tube, produced data which were used by Hallman[3] for comparison with his analytical predictions for the fully developed case. Further experimental investigations were reported later by Hallman[21] and Brown.[5] These data are in good agreement with Hallman's analysis, which is fitted well for $100<Gr^*/Re<10{,}000$ by the equation:

$$Nu=0.95\left(\frac{Gr^*}{Re}\right)^{0.28} \qquad (2)$$

Experimental results obtained by Kemeny and Somers[22] using water and oil are shown in Figure 4. However, direct comparison of these results with earlier work cannot be made because the Nusselt numbers used by Kemeny and Somers were based on the inlet fluid temperature. More recent experiments by Barozzi *et al.*[23] have been compared with numerical solutions by Collins and good agreement is reported. Evidence of transition to turbulent flow near the end of the heated section was observed for Reynolds numbers of less than 1000.

Further data can be found in the papers by Scheele and Hanratty[24] and Brown and Gauvin.[25] However, these experiments were concerned primarily with transitional flow.

The above discussion of experimental studies of laminar

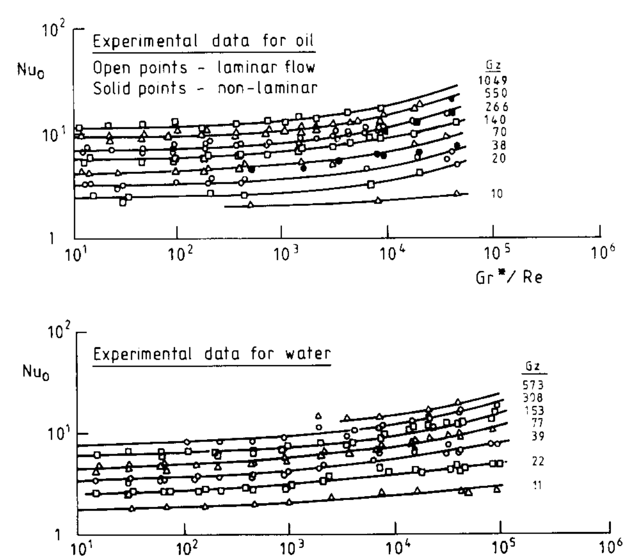


Figure 4 Plots of Nu_0 versus Gr*/Re for different values of Gz (adapted from Kemeny and Somers[22]; with permission of the American Society of Mechanical Engineers)

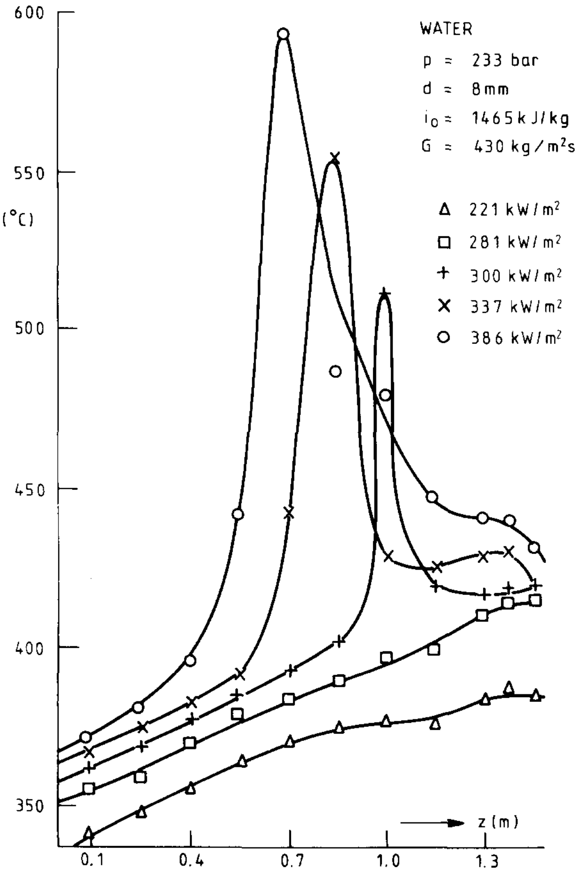


Figure 5 Localized impairment of heat transfer due to buoyancy (adapted from Shitsman[37]; with permission of Plenum Publishing Corp.)

mixed convection in vertical tubes has been restricted to the case where free convection aids forced convection. An interesting conclusion which has emerged from the present review is that virtually no laminar flow heat transfer measurements have been reported for conditions where free convection opposes forced convection (heated downward flow or cooled upward flow). For such flows the influence of buoyancy should be to impair heat transfer relative to that for conditions of pure forced convection, provided the flow remains steady and laminar. However, as a result of the work of Scheele *et al.* on buoyancy-induced instability in laminar flow, it is clear that the range of conditions available for such experiments is likely to be rather limited.

Turbulent flow mixed convection

Whereas the effects of buoyancy on heat transfer can be predicted readily for the laminar case, the situation is very different with turbulent flows: the data for such conditions show some unexpected trends (see reviews[26–28]). In configurations with forced and free convection aligned, local heat transfer coefficients significantly lower than those for forced flow alone can result. In contrast, for downward flow in heated tubes buoyancy forces cause a general enhancement of the turbulent diffusion properties of the flow, with the result that wall temperature distributions are well-behaved and heat transfer coefficients are higher than those for forced flow alone. Eventually, as the free convection component becomes more and more dominant, heat transfer for upward flow also becomes enhanced and in the limit the heat transfer coefficients for the two cases are the same.

Examples of buoyancy-induced impairment of heat transfer

In the earliest studies of turbulent mixed convection with atmospheric presure water and air,[18,29–34] there was little indication of the dramatic influences of buoyancy on heat transfer that were later to become evident in the work on fluids near the critical point[35,36] and which have provided the incentive for considerable research work in recent years.

During early development work on supercritical pressure steam generators, Shitsman[37] and Ackerman[38] found severe localized impairment of heat transfer for upward flow in heated tubes at near-critical conditions (Figure 5). The effect was initially thought to be similar to film-boiling and was, in fact, given the name "pseudoboiling." There were, however, some surprising conditions where the wall temperatures were well below the pseudocritical value.[39,40] Under such conditions the fluid must have been in a liquid-like state even in the wall-layer region.

It became apparent that the effect was due to buoyancy and not a form of film boiling when experiments with upflow were compared with those for downflow at otherwise identical conditions. Investigations of this type were first reported by Shitsman[41,42] and Jackson and Evans-Lutterodt[43] (Figure 6). Similar comparisons have since been made by a number of researchers, notably Bourke *et al.*,[44], Fewster,[45] Alferov *et al.*,[40,46,47] Watts and Chou,[48] and Bogachev *et al.*[49].

The fact that the phenomenon of localized impairment due to buoyancy is not restricted to fluids at high pressure, but can occur in liquids and gases at normal pressure, has subsequently become clear as a result of experiments by Steiner,[50] Kenning *et al.*,[51] Hall and Price,[52,53] and Fewster.[45] Kenning *et al.* found wall temperature peaks for upward flow of water at 5 bar (Figure 7) and Fewster made comparisons between upward and downward flow of atmospheric pressure water (Figures 8 and 9). The case shown in Figure 9 is of special interest because the Reynolds number is below that normally associated with turbulent flow. The measurements suggest that buoyancy-induced turbulence is present, the sudden fall in wall temperature for upward flow being due to transition.

Mixed convection at low Reynolds number has also been studied by Rouai.[54] Buoyancy-induced transition to turbulent

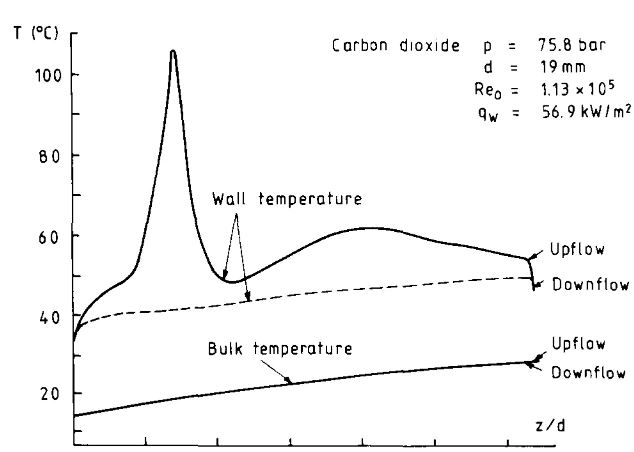


Figure 6 Comparison of upward and downward flow cases at near critical conditions (from Jackson and Evans-Lutterodt[43])

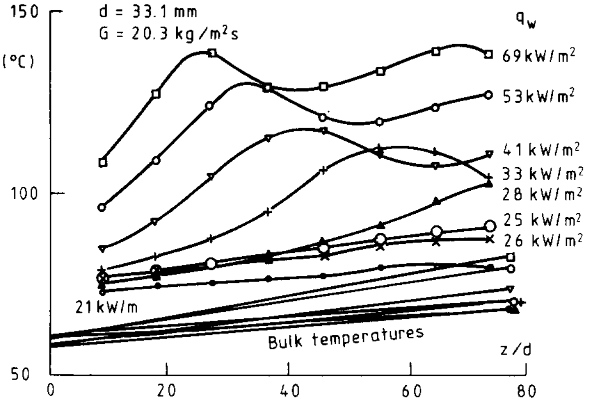


Figure 7 Buoyancy-induced impairment with water at a pressure of 5 bar (adapted from Kenning *et al.*[51]; with permission of Harper and Row)

flow was clearly observed in upward flow (Figure 10a) whereas for downward flow turbulence existed throughout the heated section (Figure 10b). In downward flow a periodic Nusselt number variation (Figure 11) suggested that small, buoyancy-induced recirculation cells might have been present. Wilkinson *et al.*[55] examined the conditions for flow reversal at the pipe wall in a buoyancy-opposed flow; otherwise the problem has received little attention.

The examples quoted above illustrate the importance of interactions between forced and free convection for flow in vertical pipes. An explanation of the mechanism involved has been given by Hall and Jackson.[56] They suggested that the dominant factor was the modification of the shear stress distribution across the pipe, with consequential change in turbulence production. The analysis of Hall and Jackson[56] has been extended by Jackson and Hall[28] to provide a general criterion for the onset of buoyancy effects applicable both for supercritical and normal pressure fluids. For influences of buoyancy on heat transfer coefficient to be less than 5% of the forced convection value, the analysis yields a limit:

$$\frac{\overline{\mathrm{Gr}}_b}{\mathrm{Re}_b^{2.7}} < 10^{-5} \tag{3}$$

Comparison of predicted criterion with experiment

In order to test the validity of the criterion, experimental data from the authors' programme of research on heat transfer to supercritical pressure carbon dioxide have been presented in terms of the parameter $\overline{\mathrm{Gr}}_b/\mathrm{Re}_b^{2.7}$. Figures 12(a) and (b) show experimental data that are consistent with the criterion (the ordinate is the ratio of the observed Nusselt number to that for forced flow in the absence of buoyancy effects but under conditions that are otherwise identical).

Alferov *et al.*[57] presented data for supercritical pressure water in terms of the ratio of calculated heat transfer coefficients for forced and free convection (Figure 13). It can be demonstrated with the aid of established correlation equations for free and forced convection that $h_{\mathrm{free}}/h_{\mathrm{forced}}$ is proportional to $\mathrm{Gr}/\mathrm{Re}^{2.46}\mathrm{Pr}^{0.5}$ raised to the power one third. Thus the criterion for negligible buoyancy suggested by the data of Alferov *et al.*[57] (Figures 13 and 14), namely $h_{\mathrm{forced}}/h_{\mathrm{free}} > 3$, can be re-expressed as:

$$\frac{\mathrm{Gr}}{\mathrm{Re}^{2.46}\mathrm{Pr}^{0.5}} < 2.4 \cdot 10^{-5} \tag{4}$$

For $10^4 < \mathrm{Re} < 10^5$, this criterion gives effectively the same classification of data as that of Jackson and Hall.

Some further data which can be used for testing the validity of the criterion have been produced by Brassington and Cairns[58] for supercritical pressure helium. Buoyancy-induced wall temperature peaks occurred over a wide rage of reduced pressure but it was found that such effects were not present

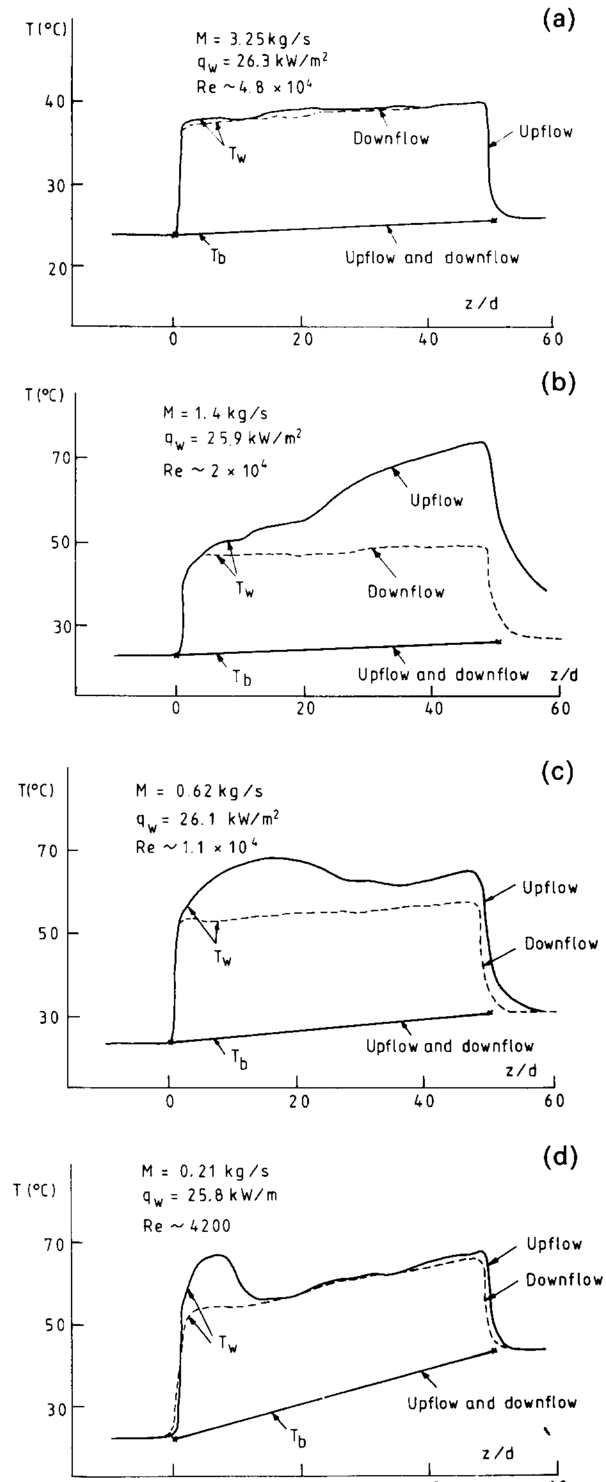


Figure 8(a–d) Comparison of heat transfer for upward and downward flow of water at atmospheric pressure in a tube of diameter 100 mm (adapted from Fewster[48])

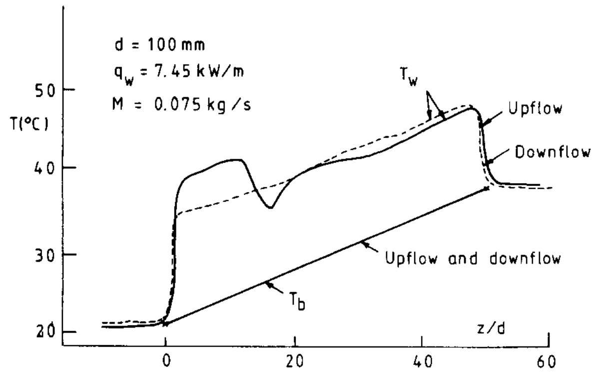


Figure 9 Buoyancy-induced transition to turbulent flow in water at atmospheric pressure at a Reynolds number of 1500 (adapted from Fewster[45])

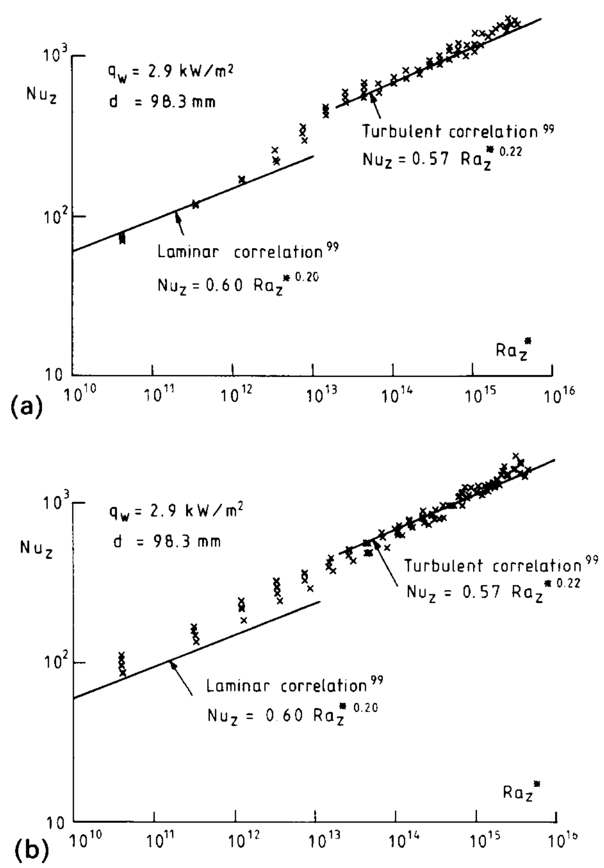


Figure 10 Correlation of low flow data for water (adapted from Rouai[54]): (a) upflow and (b) downflow

under conditions where:

$$\frac{Gr_b}{Re_b^{2.7}} < 2.4 \cdot 10^{-5} \tag{5}$$

Bearing in mind that $\overline{Gr}_b$ is somewhat less than Gr_b, the agreement with the criterion is extremely good.

Although the main sources of data concerning the onset of buoyancy effects have been investigations with supercritical pressure fluids, some work has also been done using water at pressures near to atmospheric[45,51] and with atmospheric pressure air.[59] These studies have further substantiated inequality (3) as a reliable criterion for the determination of conditions under which buoyancy effects can be neglected.

A simple equation describing mixed convection heat transfer in vertical tubes

Although the theory of Hall and Jackson was developed in order to obtain a criterion for the onset of heat transfer impairment for the case of upward flow in a heated pipe, it is also applicable to downward flow where there is heat transfer enhancement. It has been generalized[28] to provide the following simple description of the manner in which the ratio of buoyancy-influenced to buoyancy-free Nusselt numbers varies with the buoyancy parameter $\overline{Gr}_b/Re_b^{2.7}\overline{Pr}_b^{0.5}$

$$\frac{Nu}{Nu_F} = \left|1 \pm \frac{10^4 \overline{Gr}_b}{Re_b^{2.7}\,\overline{Pr}_b^{0.5}}\right|^{0.46} \tag{6}$$

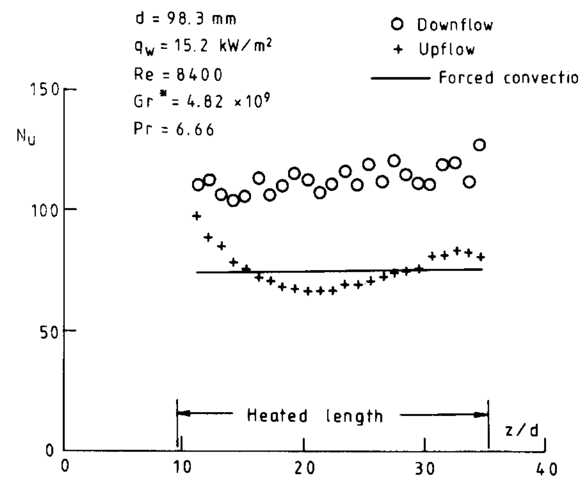


Figure 11 Nusselt number variation in mixed convection heat transfer to water (adapted from Rouai[54])

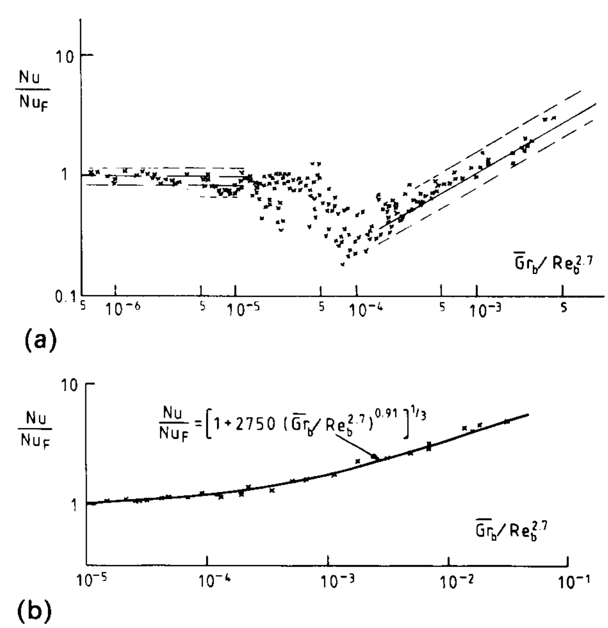


Figure 12(a–b) Mixed convection data for supercritical carbon dioxide (adapted from Fewster[48]): (a) upward flow and (b) downward flow

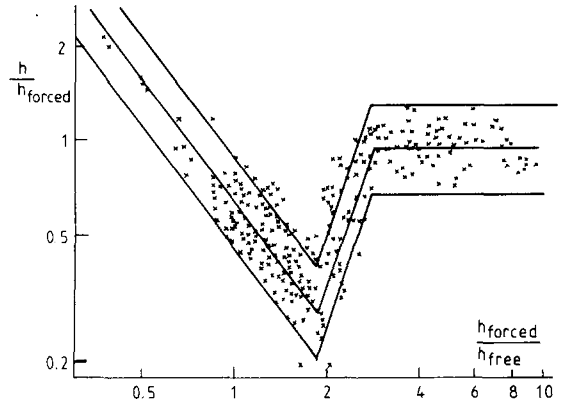


Figure 13 Correlation of data for upward flow of supercritical pressure fluids (adapted from Alferov *et al.*[57]; with permission of Scripta Technica Inc.)

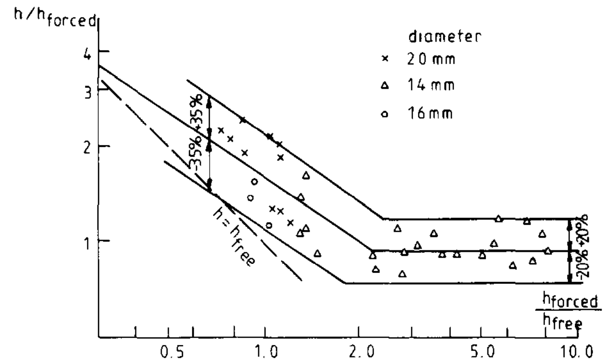


Figure 14 Correlation of downward flow mixed convection data for supercritical pressure water (adapted from Alferov *et al.*[57]; with permission of Scripta Technica Inc.)

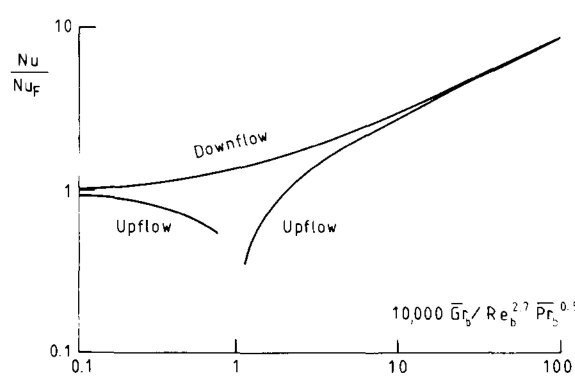


Figure 15 Theoretical prediction of general features of mixed convection heat transfer in vertical tubes (adapted from Jackson[60])

The negative sign refers to the buoyancy-aided case and the positive sign to the opposed case. It should be borne in mind that this expression was obtained using simple physical theory and empirical relationships for buoyancy-free heat transfer (for details see Ref. 28). In consequence it merely represents an attempt to predict gross trends as buoyancy effects begin to modify the forced flow significantly, however it is surprisingly consistent with observed behavior (see below).

With increase in the buoyancy parameter, an impairment of heat transfer is indicated for upward flow and an enhancement for downflow. The curves representing Equation 6 for the two cases are shown in Figure 15. It is of interest to note that for upward flow with strong enough buoyancy influence a recovery in heat transfer is indicated, leading eventually to enhancement at high values of the buoyancy parameter. Data for upward flow of air[59] and mercury[61] and for downward flow of water[62] are shown in Figures 16, 17, and 18, respectively. These show behavior consistent with that indicated by Equation 6.

Correlation of data for mixed convection

The theory referred to in the previous sections has proved to be useful for correlating data and has been employed when presenting data from the authors' program of research into heat transfer to fluids at supercritical pressure—see Figures 12(a) and (b) (earlier). It is clear that for upward flow (Figure 12a) the data do not correlate in the region where heat transfer is impaired in terms of the parameters Nu/Nu_F and $\overline{Gr}_b/Re_b^{2.7}$. However, for downward flow (Figure 12b) an excellent correlation is achieved throughout the mixed convection regime by the equation

$$\frac{Nu}{Nu_F}=\left[1+2750\left(\frac{\overline{Gr}_b}{Re_b^{2.7}}\right)^{0.91}\right]^{1/3} \tag{7}$$

The form of the above equation was chosen such that Nu becomes independent of Re_b for conditions where buoyancy is dominant (i.e. tending towards pure free convection).

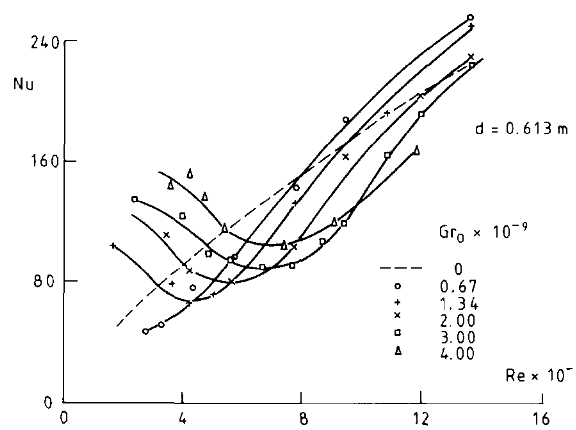


Figure 16 Effect of buoyancy on heat transfer to air at atmospheric pressure for upward flow (adapted from Byrne and Ejiogu[59]; with permission from the Council of the Institute of Mechanical Engineers)

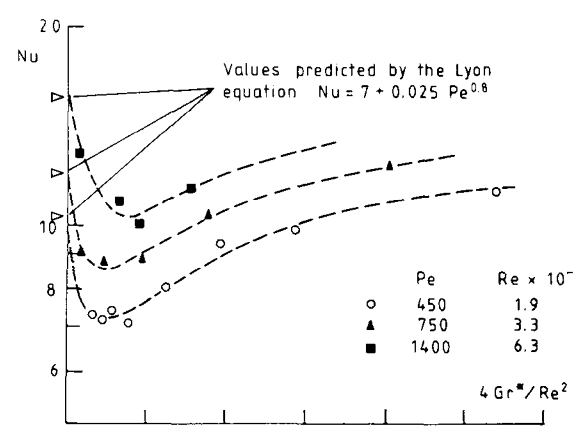


Figure 17 Heat transfer with an ascending mercury flow in a heated vertical pipe (adapted from Buhr *et al.*[61]; with permission of the American Society of Mechanical Engineers)

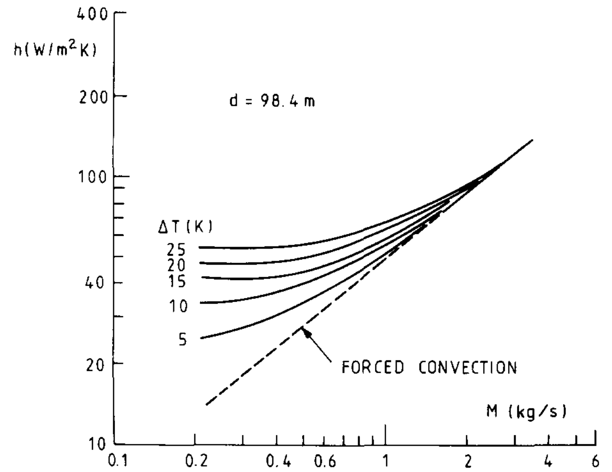


Figure 18 Effect of buoyancy on heat transfer to water at atmospheric pressure for downward flow (adapted from Jackson and Fewster[62]; with permission of Harper and Row)

The contrast between the upward and downward flow cases is interesting: for the former, buoyancy forces lead to localized effects (wall temperature peaks), whereas for the latter they do not. For upward flow impairment or enhancement of heat transfer can occur depending on the magnitude of the buoyancy force, whereas for downward flow only enhancement is found. Although the effects might seem to be anomalous when viewed without the aid of the model of Jackson and Hall, they can be seen to follow a meaningful pattern when considered in the light of the model. The general picture of mixed convection in vertical tubes embodied in Equation 6 and illustrated in Figure 15 thus proves to be surprisingly accurate.

In parallel with work on supercritical pressure fluids, the present authors and co-workers have also studied mixed convection to water and to air, both at atmospheric pressure (Jackson and Fewster[62] and Axcell and Hall[64]). Again the theoretical guidelines have proved to be useful. Jackson and Fewster used as their correlating parameter the group $\overline{Gr}_b/Re^{2.625}\overline{Pr}_b^{0.5}$. The data of Jackson and Fewster for downward flow of water in a heated tube are shown in Figure 19 and can be seen to be satisfactorily correlated. A correlation equation which fitted the data over the entire range of conditions was arrived at by adjusting the index in order to make Nusselt number independent of Reynolds number when buoyancy influences become dominant. This equation is:

$$\frac{Nu}{Nu_F}=\left[1+\frac{4500\overline{Gr}_b}{Re_b^{2.625}\overline{Pr}_b^{0.5}}\right]^{0.31} \tag{8}$$

Figure 19 also includes some early data for atmospheric pressure water.[18,32] The agreement between the equation and the data is quite reasonable when account is taken of the uncertainties involved, particularly in the values of Nu_F used for normalizing the data.

Figure 20 shows the data of Axcell and Hall[64] for downward flow of air in a heated pipe plotted on the same basis. Some measurements by Easby[65] of downward flow mixed convection to nitrogen at about 4 bar are also of interest. When compared with Equation 7 (the correlation equation developed for the supercritical pressure carbon dioxide data) they are found to be in fair agreement. This is also the case for the data of Brown and Gauvin,[25] who found enhancement of heat transfer by up to 70% in downward flow experiments on air. Further sources of data on downward flow mixed convection are Krasyakova *et al.*,[66] Ikryannikov *et al.*,[67,68] and Alferov *et al.*[57] Thus it can be seen that there is a significant body of information available concerning the enhancement of heat transfer due to buoyancy effects for downward flow in heated tubes. The observed trends are in all cases consistent with the simple model of mixed convection in vertical tubes of Jackson and Hall and can be adequately described by expressions such as Equations 7 or 8. For upward flow in heated tubes a significant amount of experimental data is also available. Whilst the trends are broadly consistent with the model, the problem of correlating data is more complicated, particularly because of complex flow development behavior.

The difficulty of correlating upward flow data in terms of purely local parameters (Figure 21) led Rouai[54] to examine conditions at wall temperature peaks. Minimum heat transfer was correlated by:

$$\frac{Nu_{min}}{Nu_F}=14.91\left[\frac{Gr^*}{Re^{3.425}\,Pr^{0.8}}\right]^{0.28} \tag{9}$$

and the distance to the first wall temperature peak by:

$$\frac{z}{d}=0.051\left[\frac{Gr^*}{Re^{3.425}\,Pr^{0.8}}\right]^{-0.8} \tag{10}$$

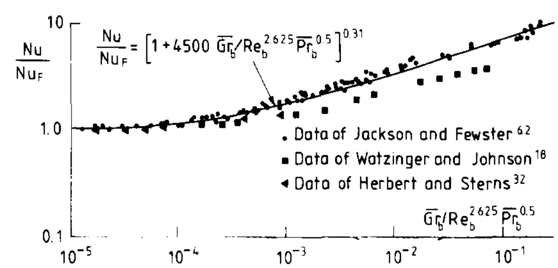


Figure 19 Correlation of mixed convection data for downward flow of water (adapted from Jackson and Fewster[62]; with permission of Harper and Row)

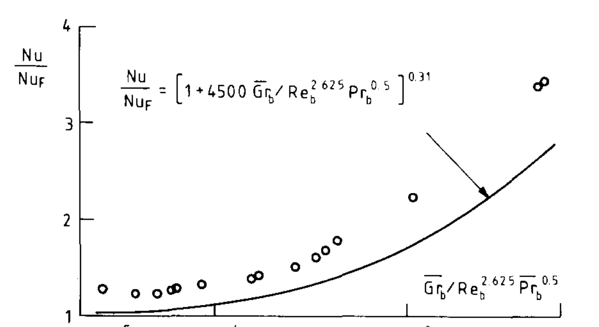


Figure 20 Comparison of downward flow correlation with data for air (adapted from Axcell and Hall[64]; with permission of Harper and Row)

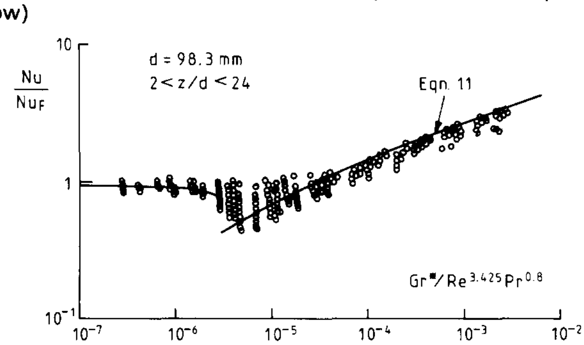


Figure 21 Upward flow data for water at atmospheric pressure (from Rouai[54])

Rouai also compared his data with a refinement of Hall and Jackson's model which accounts for the influence of heat transfer on the buoyant layer.[54,69] An implicit expression for $\mathrm{Nu}/\mathrm{Nu}_F$ results:

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_F}=\left[\left|1\pm 8\times 10^4\frac{\mathrm{Gr}^*}{\mathrm{Re}^{3.425}\,\mathrm{Pr}^{0.8}}\left(\frac{\mathrm{Nu}}{\mathrm{Nu}_F}\right)^{-2}\right|\right]^{0.46} \tag{11}$$

It should be noted that this correlation gives a discontinuity in Nusselt number for heated upward flow when $\mathrm{Gr}^*/\mathrm{Re}^{3.425}\,\mathrm{Pr}^{0.8}\sim 3\times 10^{-6}$.

Numerical studies of turbulent mixed convection in vertical tubes

The various applications of turbulence closures to mixed convection are considered under the following headings:

(i) Prescribed eddy diffusivity models
(ii) Mixing length models
(iii) One-equation transport models
(iv) Two-equation transport models
(v) Higher order models

The categorization above does not convey the complete picture because of the absence of any reference to the flow formulations adopted (whether fully developed or developing thermal-hydraulic conditions are assumed to prevail). It is well established that very long flow development lengths occur in regions of the ascending mixed convection regime and therefore the validity of studies using even the most refined turbulence models cast in a fully developed formulation can be seriously compromised.

The most general form of the mean flow governing equations used in the works reviewed below are the time-averaged momentum and energy equations written in the "thin shear" (or "boundary layer") formulations. Thus, in cylindrical polar coordinates:

Momentum

$$\frac{1}{r}\frac{\partial}{\partial r}(rVW)+\frac{\partial}{\partial z}(W^2)=-\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}z}+\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\nu\frac{\partial W}{\partial r}-\overline{vw}\right)\right]+[1-\beta(T-T_{\mathrm{ref}})]g_z \tag{12}$$

where

$$g_z=\begin{cases}-g & \text{for ascending flow}\\ +g & \text{for descending flow}\end{cases}$$

Energy

$$\frac{1}{r}\frac{\partial}{\partial r}(rVT)+\frac{\partial}{\partial z}(WT)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\nu}{\mathrm{Pr}}\frac{\partial T}{\partial r}-\overline{vT'}\right)\right] \tag{13}$$

The equations are written in the Boussinesq approximation: density variations are neglected except in the body force term of the momentum equation where a linearized function of temperature is employed. The models in categories (i) to (iv) above make recourse to the concept of turbulent viscosity by which Reynolds stress appearing in the momentum equation is related to the mean velocity gradient:

$$-\overline{vw}=\nu_t\frac{\partial W}{\partial r} \tag{14}$$

Similarly, the turbulent heat flux in the energy equation is related to the mean temperature gradient:

$$-\overline{vT'}=\frac{\nu_t}{\sigma_t}\frac{\partial T}{\partial r} \tag{15}$$

The usual practice is to set the turbulent Prandtl number to a constant value, $\sigma_t=0.9$. The different strategies adopted to determine turbulent viscosity, ν_t, are vital to the success (or lack of it) experienced in the application of turbulence models to turbulent mixed convection: it will be seen below that the simpler models fail to capture even the general trends of heat transfer impairment and enhancement.

(i) Prescribed eddy diffusivity models

The "eddy diffusivity" approach prescribes turbulent viscosity as a function of postion in the flow without any direct reference to local features of either the mean field or time-averaged turbulence field. Tanaka *et al.*[70] examined turbulent mixed convection tube flows using a modification of Reichardt's[71] eddy diffusivity model. The wall temperature distributions for a vertical heated tube computed by Tanaka *et al.* were opposite to observed behavior, showing heat transfer enhancement for ascending flow and impairment for descending flow. It was concluded that "the theory is not sufficient to estimate heat transfer coefficients." The work of Tanaka *et al.* is the first of a number of numerical studies to be reviewed in which the developing flow formulation of Equations 12 and 13 is not employed.

(ii) Mixing length models

In the Prandtl–Taylor mixing length hypothesis (MLH) it is postulated that ν_t may be expressed in terms of a mixing length, l_m, and the mean velocity gradient:

$$\nu_t=l_m^2\left|\frac{\partial W}{\partial r}\right| \tag{16}$$

Numerous modifications to the original prescription of l_m ($l_m=\kappa y$) have been proposed (see the discussion of Launder and Spalding,[72] for example). A modification that has found widespread application is due to van Driest[73] in which allowance is made for the damping effect of a solid boundary upon turbulence in the vicinity of the boundary:

$$l_m=\kappa y[1-\exp(-y^+/A^+)]\qquad(\kappa=0.4;\ A^+=26.0) \tag{17}$$

Malhotra and Hauptmann[74] implemented the van Driest mixing length model in a fully developed mean flow equation set. Computed wall temperature distributions for heated ascending and descending flow demonstrated the correct trends, indicating heat transfer impairment for upflow and enhancement for downflow. Comparison with the wall temperature data of Jackson and Evans-Lutterodt[43] for carbon dioxide at near-critical conditions showed good agreement for descending flow but poor quantitative agreement for ascending flow (Figure 22), although the highly variable fluid properties of the experiment represent a considerable added complication.

Walklate[75] tested four mixing length models against the data for heated upflow of air with uniform wall flux obtained by Carr *et al.*[76] The data show significant influences of buoyancy, demonstrating velocity profile inversion in three of the four test case and a marked reduction in the level of Reynolds stress. The four mixing length models used by Walklate represent various modifications of the van Driest formulation. Walklate used the developing flow equations of the form given above, thus eliminating an uncertainty present in the work of Tanaka *et al.*[70] and Malhotra and Hauptmann.[74] Comparison was made with the bulk parameter data (Stanton number and local friction coefficient) and profile measurements of Carr *et al.* Discrepancies between calculated and measured bulk parameters were large, and computed velocity, temperature and Reynolds stress profiles were in poor agreement with the experimental data.

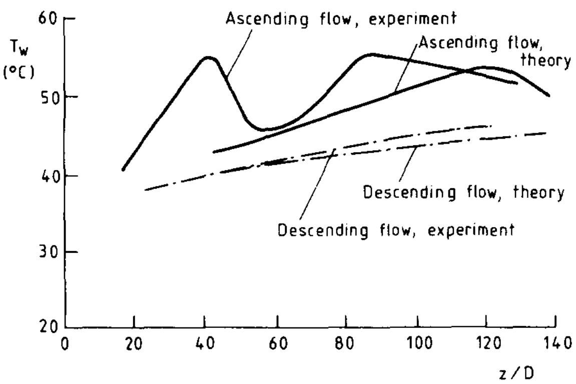


Figure 22 Comparison between calculations of Malhotra and Hauptmann[74] and data of Jackson and Evans-Lutterodt[43] for supercritical pressure carbon dioxide (adapted from Malhotra and Hauptmann[74]; with permission of Harper and Row)

An early hybrid work by Hsu and Smith[27] combined an expression for near-wall turbulent viscosity due to Deissler[77] with the mixing length model for the outer region. The calculations indicated only enhancement of heat transfer for heated ascending flow, a result that is again contrary to observed behavior. The likely cause of this result is the Deissler near-wall formulation which, although strictly not a prescribed eddy diffusivity model as defined earlier, nonetheless does not correctly reflect turbulence production. It should be added that fully developed flow was assumed in this early attempt to calculate turbulent mixed convection.

(*iii*) *One-equation transport models*

Studies of turbulent mixed convection in vertical tubes have been performed using turbulence models that employ the turbulent viscosity concept but which form ν_t from turbulence quantities and include transport effects. The basis for introduction of turbulence quantities into an equation for ν_t is the Prandtl–Kolmogorov formula in which the square root of turbulent kinetic energy, k, forms the velocity scale in the constitutive equation. Thus,

$$\nu_t = k^{1/2} l_t \qquad (18)$$

Models in which a transport equation for one of the scales (in practice the velocity scale via k-transport) is employed are known as "one-equation" models. Axcell and Hall[64] applied a variation of Wolfshtein's[78] one-equation model to their experimental data for heated descending air flows. An important feature of the Wolfshtein model is that it is applicable over the entire flow domain, including the viscosity-affected near-wall region and thus does not require the specification of "wall functions" to bridge that region. Axcell and Hall's calculations were qualitatively correct, showing enhancement of heat transfer with respect to forced convection. The predicted enhancement was, however, considerably less than that found experimentally (Figure 23). It was concluded that the one-equation model does not offer any advantage over a mixing length model in mixed convection calculations. Again there are discrepancies between the thermal-hydraulic conditions of the experiment and the modeled problem and a fully developed condition was assumed in the analysis (although rapid thermal-hydraulic development in descending mixed convection flows makes the assumption far less limiting than in ascending flows where in many cases it is wholly inappropriate).

(*iv*) *Two-equation transport models*

"Two-equation" models have been applied with some success to turbulent mixed convection. These incorporate transport effects on both the velocity and length scales appearing in the constitutive equation for ν_t. The turbulence quantities most often selected to form the scales are the turbulent kinetic energy, k, and its rate of dissipation, ε, as evidenced by Shih's literature surveys.[79,80] The length scale is obtained as $k^{3/2}/\varepsilon$, and the Prandtl–Kolmogorov formula takes the following form:

$$\nu_t = C_\mu k^2/\varepsilon \qquad (19)$$

Walklate[75] tested three versions of the $k\sim\varepsilon$ model with developing flow formulations against data of Carr *et al.*;[76] the first model examined was a standard "high-Reynolds-number" model (Launder and Spalding[81]) in which the k and ε transport equations were solved for the region $y^+ \geqslant 30$, analytical wall functions being employed to bridge the near-wall region. The other two models were variants of a *partial* low-Reynolds-number treatment in which a damping term was applied to the expression for ν_t (Equation 19) following Jones and Launder.[82,83] The forms of the k and ε equations were unaltered from the high-Reynolds-number version (although k-transport was solved over the entire flow). Walklate found that, as a group, the $k\sim\varepsilon$ models performed better than the mixing length models in computing turbulent mixed convection. Heat transfer calculated using the partial low-Reynolds-number models showed relatively good agreement with the data of Carr *et al.*; poorer agreement was evident when the high-Reynolds-number model was applied. These results were supported by comparisons with the flow profile measurements of Carr *et al.*; the general picture to emerge was that improvement could be gained by the use of partial low-Reynolds-number modifications. However, no model yielded high quantitative accuracy. Walklate made a single test of a partial low-Reynolds-number model against one of the velocity profiles measured by Axcell and Hall[64] for descending flow but found that agreement with data was poorer than that found in the comparisons made with the ascending flow profile data of Carr *et al.*

Abdelmeguid and Spalding[84] combined a standard $k\sim\varepsilon$ model with (unspecified) wall functions in a developing flow solution scheme. Computed results demonstrated the correct trends in mixed convection heat transfer, i.e. for heated upflow, impairment at low Grashof number was succeeded by enhancement at high Grashof number and, for heated downflow, enhancement was found for all values of Grashof number (Figure 24). There was no comparison with experimental data for bulk parameters; however, reasonable agreement was obtained with the velocity and temperature profile measurements of Buhr *et al.*[61] for heated ascending flow of mercury.

Thus, from the results of Walklate[75] and Abdelmeguid and Spalding,[84] it would appear that the $k\sim\varepsilon$ model offers an improvement over simpler models in the calculation of turbulent mixed convection, although there are significant discrepancies

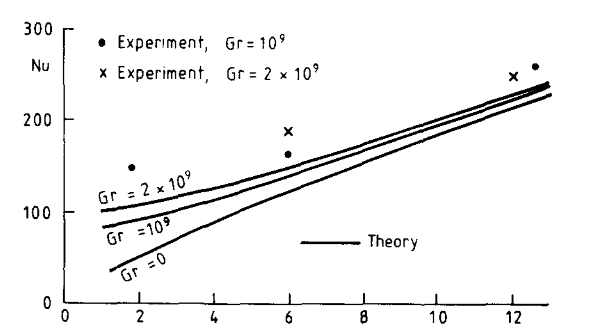


Figure 23 Comparison between experimental data and theory for downward flow turbulent mixed convection (adapted from Axcell and Hall[64]; with permission of Harper and Row)

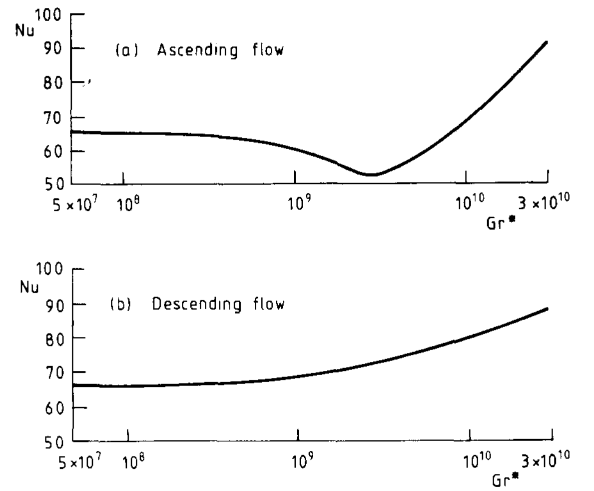


Figure 24 Calculated variation of Nu with Gr* for Re=25,000 (adapted from Abdelmeguid and Spalding[84]; with permission of Cambridge University Press)

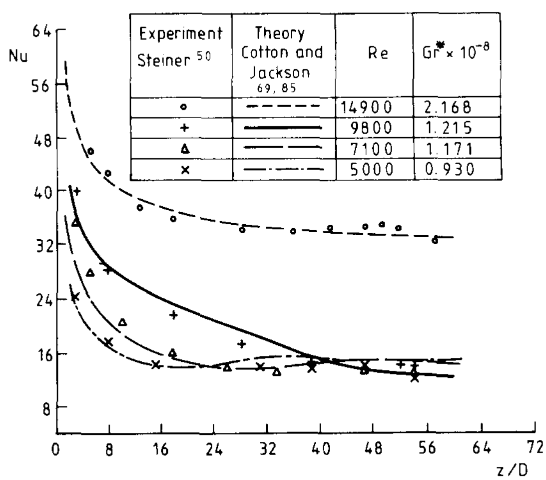


Figure 25 Comparison between calculations of Cotton and Jackson[69,85] and data of Steiner[50] for ascending air flow (adapted from Cotton and Jackson[69,85]; with permission of the American Society of Mechanical Engineers)

between computational results and experimental data. The likely cause of these discrepancies lies in the treatment of the near-wall region: Hall and Jackson[56] identified the importance of deviations from "universal" behavior in this region in determining levels of mixed convection heat transfer and Walklate found that agreement with experimental data was improved by the adoption of a partial low-Reynolds-number treatment.

Cotton and Jackson[69,85] examined the performance of a low-Reynolds-number $k \sim \varepsilon$ model due to Launder and Sharma[86] (a minor re-optimization of the Jones and Launder[82,83] model) against a wide range of mixed convection data for air flows. It was found that the full low-Reynolds-number treatment cast in the developing flow formulation reproduced to good accuracy both the experimental heat transfer and (where available) velocity, Reynolds stress and temperature profile measurements of Carr *et al.*,[76] Byrne and Ejiogu[59] and Steiner[50] (all ascending flow) and of Easby[65] (descending flow with moderate buoyancy influence). Figure 25 shows an example of the results obtained by Cotton and Jackson: the heat transfer data of Steiner are computed accurately over a development length of approximately sixty tube diameters. Good agreement was also obtained with the velocity profiles of Axcell and Hall[64] for descending flow at high levels of buoyancy, but predictions were in less than complete accord with heat transfer data, a feature that is examined further below. Polyakov and Shindin[87] have recently published experimental data showing heat transfer, velocity, temperature and turbulence parameters in heated upward flow, and the present authors intend to make comparisons with theoretical predictions as part of their continuing programme of research.

It was noted above that the data of Axcell and Hall were not computed satisfactorily by Cotton and Jackson using the Launder and Sharma model; results presented in Refs. 69 and 85 show that the calculations return considerably higher levels of heat transfer than those measured experimentally (Figure 26). Recent work by the authors[88] indicates that improved agreement with data is obtained by the inclusion of an additional source term in the ε-equation proposed by Yap.[89] Further work is in progress to investigate buoyancy-induced recirculation using an elliptic solver developed by Huang and Leschziner.[90]

Two further low-Reynolds-number $k \sim \varepsilon$ studies are reviewed. Renz and Bellinghausen[91] used the Jones and Launder model to compute heat transfer to an ascending flow of a refrigerant under the conditions of an experiment by Scheidt[92] carried out near the thermodynamic critical point (and therefore with highly variable fluid properties). The correct qualitative trends of wall temperature development were found, although there were some significant quantitative discrepancies as shown in Figure 27. Tanaka *et al.*[93] compared a slight variant of the Jones and Launder model against their data for heated upflow of nitrogen and found generally good agreement between measured and calculated Nusselt numbers. Comparison with data was limited, however, and does not appear to include points in the vicinity of maximum heat transfer impairment. Tanaka *et al.* used the fully developed formulation for their calculations which is unsuitable for application to flows with high heat transfer impairment where long development lengths occur.

(*v*) *Higher order models*

"Reynolds stress" or "second moment" turbulence models do not rely upon the turbulent viscosity concept, but instead incorporate transport equations for second order velocity and temperature correlations. Launder[94] provides a comprehensive discussion of these models. The complexity of Reynolds stress models led to the development of truncated forms known as

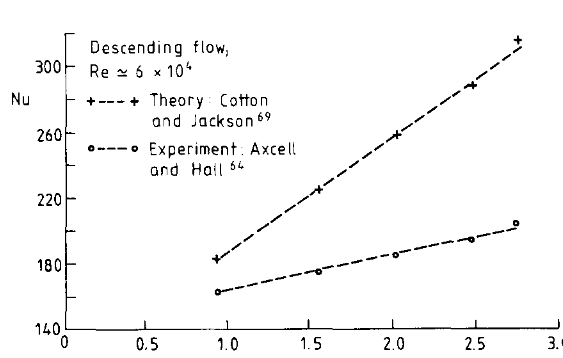


Figure 26 Comparison between calculations of Cotton and Jackson[69] and data of Axcell and Hall[64] for descending air flow (re-drawn from Cotton and Jackson[69]; with permission of the American Society of Mechanical Engineers)

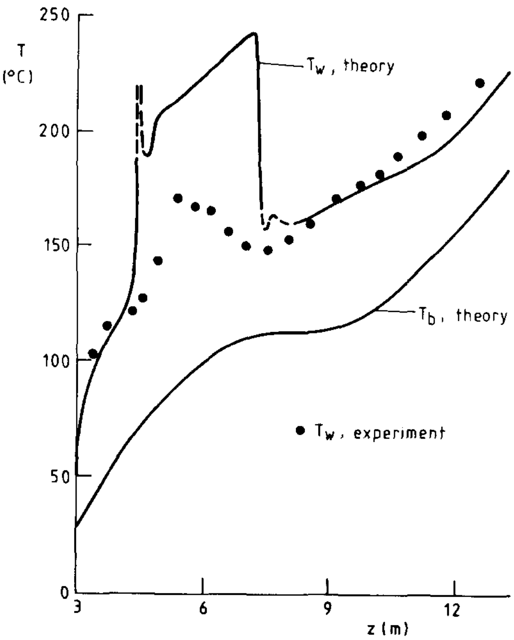


Figure 27 Comparison between calculations of Renz and Bellinghausen[91] and data of Scheidt[92] for ascending refrigerant flow (adapted from Renz and Bellinghausen[91]; with permission of Harper and Row)

"algebraic stress models" (ASMs). Reynolds stress models and ASMs have found application in complex flows; however, as shown by Launder,[95] the ASM formulation reduces to $k \sim \varepsilon$ form in thin shear flows. To and Humphrey[96] applied low-Reynolds-number $k \sim \varepsilon$ and ASM formulations to free convection along a heated vertical plate but found only slight differences between the performance of the two models. De Lemos and Sesonske[97] applied an ASM to mercury mixed convection tube flow and found qualitative agreement with data.

Finally, the direct interaction of buoyancy and turbulence via the fluctuating density-velocity correlation is considered. Abdelmeguid and Spalding[84] presented results neglecting buoyant production and reported that tests indicated that inclusion of such terms in the k and ε equations had no significant effect upon their results. The results of Cotton and Jackson[85] confirm this finding, at least to the extent that the buoyant production terms were found to exert at most a second order influence. Petukhov and Medvetskaya[98] adopted a somewhat unusual two-equation turbulence model in which equations were formulated for turbulent kinetic energy and mean square enthalpy fluctuations with buoyant production terms included but convection and diffusion neglected. Bulk parameter calculations were consistent with experimental data but no comparison with profile measurements were made. Further work is necessary in order to clarify the importance of direct interaction terms.

Conclusions

In laminar mixed convection heat transfer is enhanced in heated upward flow and impaired in heated downward flow. The problem is amenable to calculation, although transition to turbulent flow may occur earlier than for forced convection or free convection alone.

Turbulent mixed convection heat transfer to moderate Prandtl number fluids is dictated by changes in turbulent diffusion. In heated upward flow heat transfer is impaired with modest buoyancy and enhanced with high buoyancy. It is not possible to correlate with precision heat transfer in the region of impairment using local parameters. In contrast heat transfer levels in heated downflow increase monotonically with increasing buoyancy and have been correlated successfully in terms of local variables.

Turbulent mixed convection may often be calculated accurately using turbulence modeling techniques provided that a developing flow solution is used and that the turbulence model allows for changes in both turbulence velocity and length scale. "Low-Reynolds-number" models should be used, permitting solution up to the heated surface; the shear stress may vary significantly close to the wall and wall functions based on uniform shear are not applicable.

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